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EDITED BY
W. H. METZLER

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SPECIAL DEVICES IN TEACHING GEOMETRY.

By PAUL NOBLE PECK.

Since the colleges have adopted a more or less uniform standard of entrance requirements, necessitating a high degree of efficiency on the part of the candidates for admission, the burden of preparation falls upon the secondary schools. This raising of the standard on the part of the colleges frequently sorely taxes these schools by requiring a greater amount of work in preparatory subjects without any proportionate increase in the time supposed to be devoted to such preparation. Many of our high schools and private preparatory schools today are doing much of the work that, a decade ago, was considered the function of the first-year college classes. To complete the additional work imposed by this increase in the entrance requirements without slighting any of the subjects is by no means a simple task. The work must be, to a certain extent, condensed and simplified, or special devices resorted to as a means of arousing the student's interest or directing and conserving his energies.

In the multitudinous array of subjects in which entrance examinations are now offered, mathematics, since it is required of all students, occupies a most prominent place. It must be said of many of the preparatory schools, that they are meeting in a satisfactory manner the demand made upon them by the colleges and technical schools for well-trained students in this subject.

The object of this paper is not to criticise the general training in mathematics given in the secondary schools, but rather to offer, in the special topic of geometry, suggestions which the writer trusts may be of assistance in lightening the constantly increasing burden of preparation.

From my personal experience in the class-room I shall briefly summarize the most important deficiencies of the average college freshman class in geometry.

There is a marked tendency on the part of the student to keep close to the text, to draw the figure and to letter it as it is in the book. This carries with it the effort to memorize, in part, at least, the exact words of the proof, particularly in all propositions relating to or in any way involving the theory of limits. I have frequently requested a student who has given a letter-perfect book proof to erase his figure and to draw it again in a different position and with other lettering, and in nearly all cases with the same result—the pronounced discomfiture of the student due to his inability to proceed! Such cases of course are the result of memorizing the text and figure without any clear idea of the relation of the particular proposition to the others that have preceded it. The student who attempts by this means to go through the ordeal of the day's recitation is invariably the one who sooner or later informs you, in a sudden burst of confidence, that he does not like mathematics but his particular forte is French or German!

Another serious fault is inaccuracy in definitions and carelessness in constructions. When a student is told that he should not memorize his proof, but rather make it his own before giving it, he usually goes to the other extreme in his definitions and constructions, with occasionally surprising results. The three following will illustrate my point. One student, upon being asked to define a right angle, stated that it was the angle formed by two perpendicular lines, and when further asked when two lines are perpendicular, replied, "When they are at right angles." Another student, a middle-aged man, was requested to let fall a perpendicular to a line from an outside point. His reply, given in all seriousness, was, "Let go of it"! A third student was asked to draw a tangent to a circle from a point without the circle, and the amazing reply was, "It can't be done." When

asked why the problem was, to his way of thinking, impossible, he surprised the class by explaining that "You can't draw a tangent to a circle if the circle isn't there"! Evidently, from the very nature of their replies, these students had no clear understanding of the peculiar function of geometry and were not impressed with the need of accuracy in statement or construction.

The fault here lies in the very beginning of the students' training in the subject. Had they acquired the proper geometrical view-point, such answers, of course, would not have been given. So then, the responsibility for the student's proper progress in geometry rests with the preparatory schools. If a student complete his course with but a hazy conception of what is to be expected of him when he continues his subject in college, he is doomed at best to trail along at the end of his class and may consider himself fortunate if he is rated among the "simply passed."

The next point I shall consider is the tendency of students, when given a choice between numerical problems and general theorems, to select the former. This has been such a general experience in my freshman classes that I am forced to conclude that the average student prefers the solution by formula to any that would in the slightest degree tend to test his ability to reason for himself. It is so much easier to substitute numerical values for the letters in the formula and then turn the crank and await results! Not that I undervalue the numerical problem, for I believe that it is an essential in the application of theory to practice, but this preference on the part of the student is an evidence of lack of confidence in his reasoning powers and should be early overcome.

The average freshman, I have discovered, has little or no knowledge of the historical side of mathematics and is completely bewildered if asked to explain, for example, the difference between the Pythagorean Proposition and the Pons Asinorum. There may be advanced the argument that the history of mathematics is peculiarly a topic for study in the college course. Certainly the preparatory school is burdened and has scarcely enough time as it is, without being called upon to teach this additional topic in the already long list of subjects

required for college entrance. It seems to me, however, that much of the important historical data can be given the student at the time he meets with propositions of historical interest, and in this way he is enabled to absorb the valuable facts without much conscious effort.

I shall now briefly discuss a few of the faults in the method of teaching geometry in vogue in certain schools. There are many teachers who are in the habit of reading out the theorems to students as they are sent to the board, one after another, and by the time the third student is reached, the first one is back again with the request that his theorem be restated to him. This is apt to be followed by similar requests from other students and the result is a general confusion lasting sometimes for five or ten minutes. The loss of time thus occasioned is chiefly responsible for another and a graver fault. I refer to the acceptance, by teachers who are anxious to finish the assigned lesson in the period usually allotted to it, of proofs by "previous propositions." Is there a teacher who has not had pupils eager to conceal their ignorance of the particular lemmas required in the demonstration of their propositions by falling back upon that much-worn and overburdened phrase, "by a previous proposition"? If so, I have yet to meet him. To the lazy pupil this is a "thank-goodness," an oasis in a long and dreary desert! And yet there are teachers who not only permit their students to recite in this slip-shod manner, but I know of one instance where students were actually required to remember that a particular statement was true by section 245, for instance, or by paragraph 317, and this without even referring back to the section or paragraph quoted to familiarize themselves with the wording therein! What is the natural consequence of such a method? Aside from the reflection upon the teacher, it engenders carelessness of the worst sort in the pupil and if persisted in can result only in failure.

Another fault frequently encountered is the failure of the student to connect the subject matter of today's lesson with that of yesterday. This lack of continuity is a serious matter particularly in geometry and should have special attention.

I have mentioned a few of the deficiencies of students as they come to me from the high school and preparatory schools and

also the more important faults in the method of teaching used in some institutions. What steps shall be taken to remedy these conditions? It is my purpose to describe to you briefly a few devices that I am using in my own classes with excellent results. With these time is saved, confusion eliminated, accuracy of statement and a clearer understanding of the continuity of geometrical proof obtained, and in every case the student's interest is aroused and he is made to think for himself.

In the first place I have introduced the card catalogue into geometry. The text of each proposition is put on a card together with its section number as given in the text-book, and a complete index of the lemmas employed in each proposition is made by section number and filed on cards. This is for the teacher. A record-sheet is prepared covering the assignment for the day. At the top of this sheet is the record of attendance. Then follows a list of the assigned propositions and under each of these is the list of lemmas employed in its proof. All of this list is by section number and each number is followed by two blank spaces, one to be filled in with the name of the student and the other with his mark which should be recorded as soon as the recitation is finished.

In assigning the propositions to the class, I call upon the first student, hand him the card with his theorem on it and send him to the board. The card contains nothing but the text of the theorem and the section number. While he is going to the board, the next pupil is called and sent to the board with his card. This is continued for the entire assignment. While the students to whom assignments have been made are at the board drawing the figures and arranging the details of their proofs, the rest of the class can be quizzed on definitions or from charts, which I shall mention shortly. In this way every available minute is utilized and every member of the class participates in some way in the recitation each day. This in itself is quite an important detail. When the students at the board have finished drawing their figures, they one by one return the cards to the desk and take their seats. They are then called upon for the recitations, beginning with the one to whom has been assigned the most fundamental lemma. When he has finished the student who is responsible for the theorem employing this lemma is called upon

and after him the student having the proposition next higher in order and so on until the main proposition is reached. When the proof of this theorem is given, there is no need for the student to fall back upon the uncertain "previous proposition" phrase for the previous propositions are all before the class and have been discussed. This gives the class a constant review of all the text in the way it is most needed, by showing in just what way the propositions are related and upon what they depend and why this dependence exists.

I have referred to certain charts used in quizzing the class. The preparation of these I believe to be the most valuable work the student can do and by this means he acquires a most intimate knowledge of all the relations of the geometrical propositions. The student is required to prepare an index similar to the one mentioned above, showing by section number every lemma used in the proof of each proposition. Corollaries and definitions are not included, only the general propositions being used. After this has been properly verified, a geometrical tree is prepared for each book. The texts of the fundamental propositions, that is, those not requiring lemmas, are written at the bottom of a standard size piece of cardboard and enclosed in a rectangular frame. The section number corresponding to the one in the book is attached, together with the proper figure lettered as in the book or otherwise at the teacher's pleasure. Above these are placed the propositions requiring the fundamental theorems in their proof, and over these are still others requiring the second set and so on as the tree grows. Finally, at the top of the chart, in the case of the first book, are the last propositions of that book. In the charts of the other books, at the top, will be found the propositions of the particular book for which the chart is prepared and under these the dependent propositions. These framed texts are properly connected by straight lines showing the dependence of each proposition on every proposition subordinate to itself. Thus the student has before him on a single chart not only every relation between the propositions of a particular book, but the relations of all the propositions that have preceded these, as well. This gives him the entire chain and the continuity is so impressed upon him that he cannot fail to understand at a glance the inter-relation between the theorems.

On the historical side, my plan has been to assign to individual members of a class topics of interest to be carefully prepared and read to the class. With each paper is required a complete bibliography. In this way the student's interest is aroused and even the student who does not care for mathematics begins to find some pleasure in his work and to hope for the best when the examinations come!

It has been my experience in the use of all these devices that the difficulties mentioned in the first part of this paper have, in a large measure, been overcome and the work has been presented to the student in a pleasing manner and one calculated to stimulate an earnest effort on his part.

They are therefore offered in the hope that they may assist other teachers as they have the writer.

THE GEORGE WASHINGTON UNIVERSITY,
WASHINGTON, D. C.

DISQUIETUDE.

As thoughts possess the fashion of the mood
That gave them birth, so every deed we do
Partakes of our inborn disquietude
Which spurns the old and reaches towards the new.
The noblest works of human art and pride
Show that their makers were not satisfied.

For looking down the ladder of our deeds,
The rounds seem slender; all past work appears
Unto the doer faulty; the heart bleeds,
And pale Regret comes weltering in tears,
To think how poor our best has been, how vain,
Beside the excellence we would attain.

—HENRY ABBEY.